

# IMPRINTS OF GALAXY CLUSTERING EVOLUTION ON $\Delta T/T$

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## ABSTRACT

The nonlinear evolution of matter clustering induces temperature anisotropies in the cosmic microwave background. The fluctuations in temperature are produced at a relatively low redshift,  $z \lesssim 10$ , and consequently are not affected by the uncertain reionization processes that occurred at earlier epochs. The amplitude of the effect depends on the evolution model assumed for galaxy clustering. For plausible parameter choices, the temperature fluctuations are in the range  $10^{-6} \lesssim \Delta T/T \lesssim 10^{-5}$ . Observed limits on  $\Delta T/T$  would be violated if nonlinear clustering on present cluster scales commenced prior to  $z \sim 10$ .

*Subject headings:* cosmic background radiation – cosmology

## I. INTRODUCTION

Most of the studies on the anisotropy expected in the temperature of the Cosmic Microwave Background (CMB) have been based on linear density perturbations. Comparatively little attention has been given to additional effects that can be generated during the nonlinear phase of matter clustering evolution: the Sunyaev-Zeldovich effect relevant on small angular scales (Scaramella et al. 1993), bulk motions of gas (Ostriker and Vishniac 1986; Vishniac 1987; Hu, Scott and Silk 1994) and time-varying gravitational potentials. The formalism to calculate the temperature anisotropies generated by time-varying potentials during the nonlinear phase has been developed elsewhere (Martínez-González et al. 1990) for a flat background. Provided that one considers scales much smaller than the horizon and non-relativistic matter peculiar velocities, it is possible to describe the gravitational field in terms of a single potential  $\phi(t; \mathbf{x})$  that satisfies the Poisson equation

$$\nabla^2 \phi = \frac{1}{2} \rho_b a^2 \Delta(t; \mathbf{x}), \quad (1)$$

where  $a(t)$  represents the scale factor,  $\mathbf{x}$  are comoving coordinates,  $\rho_b \propto a^{-3}$  is the background density and  $\Delta$  is the density fluctuation defined by  $\rho = \rho_b(1 + \Delta)$ . We have chosen units such that  $8\pi G = c = 1$ .

The temperature anisotropies are given by the expression

$$\frac{\Delta T}{T} = \frac{1}{3} \phi_{LS} + \mathbf{n} \cdot \mathbf{v}_{LS} + 2 \int_0^{t_{LS}} dt \frac{\partial \phi}{\partial t}(t; \mathbf{x}), \quad (2)$$

where  $\mathbf{n}$  is the unit vector in the direction of the observation and subscript  $LS$  denotes the last scattering surface. In linear theory, the gravitational potential due to the linear density fluctuations is static if  $\Omega = 1$ , so only the two first terms survive (Sachs and Wolfe 1967) to give the primary anisotropies. Unless  $\Omega < 1$ , the integrated effect along the path of the photon only contributes as the fluctuations enter the non-linear regime, to induce secondary anisotropies. The effect of second-order perturbations on the CMB has been previously studied (Martínez-González et al. 1992), and were found to contribute to  $\Delta T/T$  at a level of  $\sim 10^{-6}$  for popular models of galaxy formation. The effect of isolated structures (voids, clusters, great attractors, etc.) has also been estimated to be at a level of  $\Delta T/T \sim 10^{-7} - 10^{-6}$  (Rees and Sciama 1968; Thompson and Vishniac 1987; Kaiser 1982; Nottale 1984; Martínez-González and Sanz 1990). Little attention has been paid to the intermediate regime of weak clustering. The effect of clustered matter on the CMB in the very non-linear regime has been estimated using N-body simulations, for isolated structures (van Kampen and Martínez-González 1991) and for the hot dark matter (HDM) scenario (Anninos et al 1991), where effects of the order of  $10^{-6}$  and  $10^{-5}$  are found, respectively (in the latter paper, they also include Thompson scattering).

## II. THE NONLINEAR GRAVITATIONAL EFFECT

### *a) The effect of a time-varying potential*

We consider a flat Friedmann dust model to represent our universe from recombination ( $z \simeq 10^3$ ) to the present time ( $z = 0$ ). The microwave photons are propagated since the last scattering surface ( $z \simeq 10^3$ ) in any direction and are affected by the gravitational potential due to the density fluctuations which are evolving from the linear regime to the nonlinear one. We are interested in the effect of nonlinear fluctuations generating anisotropies on the CMB. We calculate this effect assuming a 2-point density correlation function that can be justified from observations at the present time and take the time evolution from numerical simulations.

We wish to calculate the effect of nonlinear density fluctuations operating at late times (typically  $z \lesssim 6$ ) on scales much smaller than the horizon. In this case, a non-static potential  $\phi(t, \mathbf{x})$  suffices to represent the gravitational field of the matter and its effect on the photons (Martínez-González et al. 1990), whence

$$\frac{\Delta T}{T} = 2 \int_e^o dt \frac{\partial \phi}{\partial t}(t, \mathbf{x}), \quad (3)$$

$$\nabla^2 \phi(t, \mathbf{x}) = 6a^{-1} \Delta(t, \mathbf{x}), \quad (4)$$

where  $\Delta$  represents the nonlinear density fluctuations. The integral (3) is extended along the photon path:  $\mathbf{x}(t, \mathbf{n}) = \lambda(a) \mathbf{n}$  ( $\lambda(a) = 1 - a^{\frac{1}{2}}$ ) gives the distance to the photon from the observer normalized to the present horizon distance. All units have been chosen as in the previous section and the scale factor and horizon distance at the present time are unity, i.e.  $a_o = 3 t_o = 1$ .

The temperature correlation function  $C(\alpha)$  is obtained from the previous equation by averaging over all direction pairs separated by an angle  $\alpha$

$$C(\alpha) = 4 \int_0^{\lambda_e} d\lambda_1 \int_0^{\lambda_e} d\lambda_2 \frac{\partial^2}{\partial \lambda_1 \partial \lambda_2} C_\phi(\lambda_1, \lambda_2; x), \quad (5)$$

$$x(\lambda_1, \lambda_2, \alpha) \equiv |\lambda_1 \mathbf{n}_1 - \lambda_2 \mathbf{n}_2| = [\lambda_1^2 + \lambda_2^2 - 2\lambda_1 \lambda_2 \cos \alpha]^{1/2}, \quad (6)$$

where  $\mathbf{n}_1 \cdot \mathbf{n}_2 = \cos \alpha$ . Thus,  $C(\alpha)$  is given in terms of the correlation of the gravitational potential  $C_\phi(\lambda_1, \lambda_2; x)$  at two different cosmic times  $\lambda_1$  and  $\lambda_2$ .

### *b) Models for the evolution of galaxy clustering*

We do not have information from observations about the matter correlations at different cosmic times. Observations of the galaxy and galaxy cluster distributions are limited to the local universe with approximately the same cosmic time for all galaxies and clusters. However, numerical simulations of current models of galaxy formation (Cole and Efstathiou 1989; Hamilton et al. 1991) imply that there is a simple time scaling for the evolution of the matter correlation function. We will assume the following 2-point correlation function  $\xi(z_1, z_2; r)$

$$\begin{aligned} \xi(z_1, z_2; r) &= [(1 + z_1)(1 + z_2)]^{-\frac{3+\epsilon}{2}} \left(\frac{r_o}{r}\right)^\gamma, \quad r \leq r_m, \\ &= 0, \quad r > r_m. \end{aligned} \quad (7)$$

Here  $r$  represents the physical coordinate and  $r_m$  is a large-scale cut-off beyond which any correlations are assumed to be vanishingly small. The previous formula can be justified as follows. The spatial dependence is obtained from present epoch observations of galaxy clustering,  $\gamma \simeq 1.8$  and  $r_o \simeq 5 h^{-1} \text{Mpc}$  (Davis and Peebles 1983; De Lapparent et al. 1988), which extend to a scale  $r_m \simeq 15 h^{-1} \text{Mpc}$ . However, positive correlations have recently been found in the APM catalog (Maddox et al. 1990) up to angular scales of  $\sim 5^\circ$  (corresponding to length scales of  $\sim 30 h^{-1} \text{Mpc}$ , given the magnitude limit of the catalog). An equivalent signal is also apparent in the counts in cells of IRAS galaxies of amplitude  $\Delta N/N \simeq 0.5$  in cubes of  $30 h^{-1} \text{Mpc}$  size (Saunders et al. 1991). Additional evidence of clustering up to scales of  $\sim 50 h^{-1} \text{Mpc}$  comes from the distribution of galaxy clusters (Bahcall and Soneira 1983, Dalton et al. 1992, Peacock and West 1992), although the amplitude of the cluster autocorrelations is strongly biased.

The evolution of correlations in numerical simulations with scale-free spectra as initial conditions,  $P(k) \propto k^n$ ,  $-1 \leq n \leq 0$ , can be approximately described by the power law  $\xi(z, r) = (1 + z)^{-3} (r_o/r)^\gamma$  for scales  $r \leq 2 r_o (1 + z)^{-1/\gamma}$  (Cole and Efstathiou 1989) at late times, ( $z \lesssim 6$ ). This is equivalent to setting  $\epsilon = 0$  in (7) and describes the evolution of stable clustering where clusters are already formed and virialized at high  $z$ . Less radical evolution of clustering occurs if  $\xi(r, z)$  is constant with time in comoving coordinates. In this case, the clustering developed early and froze out, as would be appropriate for a biased galaxy formation model. In this case,  $\epsilon = \gamma - 3$ . It is also possible to have  $\epsilon > 0$  in models where clusters form more recently and are still evolving at present. Some N-body simulations (Melott 1992) predict  $0 \leq \epsilon \leq 3$  for scenarios with spectral indices in the range  $-3 \leq n \leq 1$ .

Wolfe (1993) has recently found that the clustering of damped Ly $\alpha$  absorption systems ( $z \approx 2 - 3$ ) in quasars is best fitted with an evolution exponent  $\epsilon = -1.2$  ( $\gamma = 1.8$ ). In what follows, we will not choose any specific value of  $\epsilon$ , but we will consider values within a broad interval ( $-1.2 \leq \epsilon \leq 2$ ) that covers the range of interest. Roughly speaking, values of  $\gamma - 3 \leq \epsilon \leq 0$  will represent different evolutionary models where clusters form very early, whereas  $\epsilon > 0$  describes late and ongoing clustering, as would be appropriate, for example, if  $\Omega \ll 1$  and  $\Omega = 1$ , respectively.

The above expression for  $\xi$  can be rewritten in terms of  $\lambda$  and comoving distance  $x$  as

$$\begin{aligned}\xi(\lambda_1, \lambda_2; x) &= [(1 - \lambda_1)(1 - \lambda_2)]^{3-\gamma+\epsilon} \left(\frac{r_o}{x}\right)^\gamma, \quad x \leq x_m, \\ &= 0 \quad x > x_m.\end{aligned}\tag{8}$$

We will test this type of 2-point correlation function in terms of the anisotropies generated in the CMB. The *ad hoc* cut-off  $x_m$  is time-dependent and represents the maximum scale where the nonlinear evolution model is appropriate:  $\xi(\lambda_1, \lambda_2; x) = \xi_c$ ;  $\xi_c \gtrsim 0.1$ . We write

$$x_m = r_o \xi_c^{-\frac{1}{\gamma}} [(1 - \lambda_1)(1 - \lambda_2)]^{\frac{3-\gamma+\epsilon}{\gamma}}.\tag{9}$$

Thus we have three phenomenological parameters to describe clustering evolution: the initial redshift for the start of nonlinear evolution  $z_m$  ( $\lambda_m = 1 - (1 + z_e)^{-1/2}$ ), the evolution parameter  $\epsilon$  and the correlation cut-off  $x_m$  at present,  $x_{mo} \equiv r_m = r_o \xi_c^{-\frac{1}{\gamma}}$ .

### c) $\Delta T/T$ for models of galaxy clustering

From the Poisson equation (4) one obtains

$$\nabla_{\mathbf{x}_1}^2 \nabla_{\mathbf{x}_2}^2 C_\phi(\lambda_1, \lambda_2; x) = 36 [(1 - \lambda_1)(1 - \lambda_2)]^{-2} \xi(\lambda_1, \lambda_2; x),\tag{10}$$

Assuming standard boundary conditions: i)  $|C_\phi| < \infty$ , ii)  $C_\phi(\infty) = 0$ , iii)  $C'_\phi(0) = 0$  and iv) continuity up to the second derivative of  $C_\phi$ , the correlation of the gravitational potential, associated to the correlation of matter given by equation (8), is unique and defined by the expression

$$\begin{aligned}C_\phi &= f g x_o^\gamma x_m^{4-\gamma} \left\{ 3[(4 - \gamma)(3 - \gamma) - 2] - (5 - \gamma)(4 - \gamma) \left(\frac{x}{x_m}\right)^2 + 6 \left(\frac{x}{x_m}\right)^{4-\gamma} \right\}, \quad x \leq x_m, \\ &= f g x_o^\gamma x_m^{4-\gamma} 2(4 - \gamma)(2 - \gamma) \left(\frac{x_m}{x}\right), \quad x > x_m,\end{aligned}\tag{11}$$

where

$$f = [(1 - \lambda_1)(1 - \lambda_2)]^{1-\gamma-\epsilon}, \quad g = -\frac{1}{5-\gamma} + \frac{3}{4-\gamma} - \frac{3}{3-\gamma} + \frac{1}{2-\gamma}.\tag{12}$$

In the next section we illustrate these results for the potential correlation,  $C_\phi(x)$ , and the temperature correlation,  $C(\alpha)$  (as given by equations 5 and 11), for several cases of galaxy clustering evolution.

## III. RESULTS

The behaviour of the potential correlation function  $C_\phi(x)$  is shown in Fig. 1 as a function of the comoving distance  $x$  for several values of the evolution parameter  $\epsilon$  and initial redshift  $z_m$ . The correlations grow with  $z$  for comoving clustering ( $\epsilon = -1.2$ ) whereas they decrease for stable clustering ( $\epsilon = 0$ ) and for more rapid evolution ( $\epsilon > 0$ ). This behaviour of  $C_\phi$  implies that the nonlinear effect on  $\Delta T/T$  depends on the initial redshift at which the nonlinear evolution of matter initially occurs for comoving clustering. For stable and faster clustering evolution, the effect will be produced at low redshift  $z \lesssim 3$  even if we extrapolate the power law evolution (eq. 8) to arbitrarily high  $z$ .

Fig. 2 shows the temperature correlation function generated by the nonlinear evolution of matter given by eq. (8) for three cases of the evolution parameter  $\epsilon = -1.2, 0, 2$ . The correlation scale is of the order of a degree and is larger for higher  $\epsilon$ . In the case of comoving clustering (fig. 2a), most of the effect is produced close to the initial redshift  $z \lesssim 10$ . However for stable clustering the temperature correlations are generated in the range  $3 \lesssim z \lesssim 0.5$ , (fig. 2b) and for  $\epsilon = 2$  in the range  $0.5 \lesssim z \lesssim 0.1$  (fig. 2c).

We also find that  $C(\alpha)$  is very sensitive to the matter correlation cut-off  $\xi_c$ . This is shown in fig. 2d,e,f for  $\xi_c = 0.287(r_m = 10 h^{-1} \text{Mpc}), 0.2, 0.1$ .

The main results that we have obtained for the different models of galaxy clustering evolution can be summarized as follows:

- (a) The temperature correlations  $C(\alpha)^{\frac{1}{2}} \gtrsim 10^{-6}$  for values of the evolution parameter  $-1.2 \leq \epsilon \leq 3$ , initial redshift  $z_m \geq 3$  and cut-off  $r_m \geq 10 h^{-1} \text{Mpc}$  at present (i.e.  $\xi_c \leq 0.287$ ).
- (b) The effect on  $\Delta T/T$  reaches a maximum for comoving clustering ( $\epsilon = -1.2$ ), decreases for stable clustering ( $\epsilon = 0$ ) and slowly increases for  $\epsilon > 0$ .
- (c) More than 80% of the nonlinear effect is produced in the interval  $z \lesssim 3$  for  $\epsilon \gtrsim 0$  whereas for  $\epsilon = -1.2$  the effect occurs near the initial redshift when comoving nonlinear evolution commences.
- (d) Considering present experimental limits on degree scales, the evolution of the correlations for comoving clustering as given by eq. (8) (with  $r_m = 10 h^{-1} \text{Mpc}$ ) must have commenced at  $z \lesssim 10$ , otherwise the present limits on  $\Delta T/T$  of  $\approx 10^{-5}$  would be violated. Extrapolating the observed galaxy correlation power law up to  $r_m = 19.4 h^{-1} \text{Mpc}$  ( $\xi_c = 0.1$ ) those limits imply that clusters must have formed later than  $z \simeq 4$  for the case of comoving evolution.

#### IV. CONCLUSIONS

We have shown that galaxy clustering evolution typically generates CMB temperature fluctuations in the range  $10^{-6} \lesssim \Delta T/T \lesssim 10^{-5}$ . For some specific models, an observable effect on degree scales can be obtained. This anisotropy is generated at redshifts below  $\sim 10$  and therefore is not erased by reionization processes that could take place in the universe at high  $z$ .

Observational limits on the anisotropies of the CMB on degree scales constrain models for the evolution of galaxy clustering and therefore of cluster formation. The maximum anisotropy is generated in the case of comoving clustering where the limits on  $\Delta T/T$  are violated if we extrapolate the observed galaxy correlation power law up to  $19.4 h^{-1} \text{Mpc}$  (corresponding to a correlation  $\xi_c = 0.1$ ) with  $z_m = 6$  or allowing  $z_m \gtrsim 10$  and keeping the cut-off at  $10 h^{-1} \text{Mpc}$  ( $\xi_c = 0.287$ ). The stable clustering model ( $\epsilon = 0$ ) or models with a faster evolution ( $\epsilon > 0$ ) contribute to  $\Delta T/T$  at a level of the order of  $10^{-6}$  whereas models that form structure at high- $z$  are constrained by the CMB observations.

It would be very interesting to calculate the nonlinear gravitational effect using more realistic models for the evolution of galaxy clustering that smoothly interpolate between the linear and nonlinear regimes. Such a calculation would clarify the nonlinear versus linear contributions. Indeed, from the numerical simulations done by Anninos et al. (1991) for a HDM model, it is seen that the nonlinear gravitational effect can be of the same order as the linear Sachs-Wolfe effect on degree angular scales.

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## FIGURE CAPTIONS

**Figure 1.** Potential correlation function  $C_\phi(x)$  as a function of the comoving distance  $x$  for three values of the evolution parameter: a)  $\epsilon = -1.2$  (comoving clustering), b)  $\epsilon = 0$  (stable clustering) and c)  $\epsilon = 2$ . Different lines correspond to the following redshifts for the start of nonlinear evolution:  $z_m = 0$  (solid), 0.5 (dotted), 1 (short-dashed), 3 (long-dashed) and 6 (dash-dotted). The correlation cut-off is assumed to be  $\xi_c = 0.287$ .

**Figure 2.** Temperature correlation function  $C(\alpha)$  as a function of the angular distance  $\alpha$ . a)  $\epsilon = -1.2$  and  $z_m = 10, 6, 3, 1$  (solid, dotted, short-dashed, long-dashed), b)  $\epsilon = 0$  and  $z_m = 6, 3, 1$  (solid, dotted, short-dashed) and c)  $\epsilon = 2$  and  $z_m = 6, 0.5, 0.1$  (solid, dotted, short-dashed). The correlation cut-off is assumed to be  $\xi_c = 0.287$  for cases a,b,c. d)  $\epsilon = -1.2$ , e)  $\epsilon = 0$ , f)  $\epsilon = 2$ , and  $\xi_c = 0.287, 0.2, 0.1$  ( $r_m = 10, 12.2, 19.4 h^{-1} Mpc$ ) for solid, dotted and dashed line respectively. The starting redshift  $z_m = 6$  is the same for cases d,e,f.